F FACTORISING QUADRATIC TRINOMIALS

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$$\begin{aligned}(x+2)(x+5) &= x^2+5x+2x+2\times 5 \qquad \text{\{using FOIL\}}\\ &= x^2+[5+2]x+[2\times 5]\\ &= x^2+[\textbf{sum} \text{ of } 2 \text{ and } 5]x+[\textbf{product} \text{ of } 2 \text{ and } 5]\\ &= x^2+7x+10\end{aligned}$$

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where a and b are two numbers whose sum is p, and whose product is q.

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So, if we want to factorise the quadratic trinomial $x^2+7x+10$ into $(x+...)(x+...)$ we must find two numbers to fill the vacant places which have a *sum* of 7 and a *product* of 10. The numbers are 2 and 5, so $x^2+7x+10=(x+2)(x+5)$.

Example 13

Self Tutor

Factorise:

a $x^2 - 7x + 12$

b $x^2 - 2x - 15$

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Factorise:

a $x^2 - 7x + 12$

b $x^2 - 2x - 15$

a We need two numbers with sum -7 and product 12.
The numbers are -3 and -4.
$\therefore \quad x^2 - 7x + 12 = (x-3)(x-4)$

b We need two numbers with sum -2 and product -15.
The numbers are -5 and 3.
$\therefore \quad x^2 - 2x - 15 = (x-5)(x+3)$

1 Fully factorise:

a $x^2 + 3x + 2$
b $x^2 + 5x + 6$
c $x^2 - x - 6$
d $x^2 + 3x - 10$
e $x^2 + 4x - 21$
f $x^2 + 8x + 16$
g $x^2 - 14x + 49$
h $x^2 + 3x - 28$
i $x^2 - 11x + 24$
j $x^2 + 15x + 44$
k $x^2 - x - 56$
l $x^2 - 18x + 81$
m $x^2 - 4x - 32$
n $x^2 + 4x - 45$
o $x^2 - 4x - 96$
p $x^2 + 4x - 96$

Example 14 **Self Tutor**

Fully factorise by first removing a common factor:

a $3x^2 + 6x - 72$ **b** $77 + 4x - x^2$

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a $3x^2 + 6x - 72$

$= 3(x^2 + 2x - 24)$ {3 is a common factor}

$= 3(x + 6)(x - 4)$ {sum $= 2$, product $= -24$

$\therefore$ the numbers are 6 and -4}

b $77 + 4x - x^2$

$= -x^2 + 4x + 77$ {writing in descending powers of x}

$= -1(x^2 - 4x - 77)$ {-1 is a common factor}

$= -(x - 11)(x + 7)$ {sum $= -4$, product $= -77$

$\therefore$ the numbers are -11 and 7}

2 Fully factorise by first removing a common factor:

a $2x^2 + 10x + 8$
b $3x^2 - 21x + 18$
c $2x^2 + 14x + 24$
d $5x^2 - 30x - 80$
e $4x^2 - 8x - 12$
f $3x^2 - 42x + 99$
g $2x^2 - 2x - 180$
h $3x^2 - 6x - 24$
i $2x^2 + 18x + 40$
j $x^3 - 7x^2 - 8x$
k $4x^2 - 24x + 36$
l $3x^2 + 18x - 81$
m $2x^2 - 44x + 240$
n $x^3 - 3x^2 - 28x$
o $x^4 + 2x^3 + x^2$

3 Fully factorise:

a $-x^2 - 3x + 54$
b $-x^2 - 7x - 10$
c $-x^2 - 10x - 21$
d $4x - x^2 - 3$
e $-4 + 4x - x^2$
f $3 - x^2 - 2x$

4 Fully factorise:

- **a** $-x^2 + 2x + 48$
- **b** $6x - x^2 - 9$
- **c** $30x - 3x^2 - 63$
- **d** $-2x^2 + 4x + 126$
- **e** $20x - 2x^2 - 50$
- **f** $-x^3 + x^2 + 2x$

5 Given that $x^2 + bx + c = (x + m)(x + n)$, factorise $x^2 - bx + c$.

1 **a** $(x+1)(x+2)$ **b** $(x+2)(x+3)$ **c** $(x+2)(x-3$

d $(x-2)(x+5)$ **e** $(x-3)(x+7)$ **f** $(x+4)^2$

g $(x-7)^2$ **h** $(x-4)(x+7)$ **i** $(x-3)(x-8$

j $(x+4)(x+11)$ **k** $(x+7)(x-8)$ **l** $(x-9)^2$

m $(x+4)(x-8)$ **n** $(x-5)(x+9)$

o $(x+8)(x-12)$ **p** $(x-8)(x+12)$

2 **a** $2(x+1)(x+4)$ **b** $3(x-1)(x-6)$

c $2(x+3)(x+4)$ **d** $5(x+2)(x-8)$

e $4(x+1)(x-3)$ **f** $3(x-3)(x-11)$

g $2(x+9)(x-10)$ **h** $3(x+2)(x-4)$

i $2(x+4)(x+5)$ **j** $x(x+1)(x-8)$

k $4(x-3)^2$ **l** $3(x-3)(x+9)$

m $2(x-10)(x-12)$ **n** $x(x+4)(x-7)$

o $x^2(x+1)^2$

3 **a** $-(x-6)(x+9)$ **b** $-(x+2)(x+5)$

c $-(x+3)(x+7)$ **d** $-(x-1)(x-3)$

e $-(x-2)^2$ **f** $-(x-1)(x+3)$

4 **a** $-(x+6)(x-8)$ **b** $-(x-3)^2$

c $-3(x-3)(x-7)$ **d** $-2(x+7)(x-9)$

e $-2(x-5)^2$ **f** $-x(x+1)(x-2)$

5 As $b = m+n$, $-b = -m-n$

Since $c = mn$,
$$\begin{aligned} x^2 - bx + c &= x^2 - mx - nx + mn \\ &= (x-m)(x-n) \end{aligned}$$